Indian Statistical Institute
Chennai Centre
M.Stat. S.Stream I Year 2015-2016
Second Semester
Semestral Examination

Design of Experiments 25.04.16

Answer as much as you can. The maximum you can score is 56.
The notation used have their usual meaning unless stated otherwise.
State clearly the results you use.
Time :- 2 hours

1. Consider a general block design with v treatments, b blocks with constant block size k and replication numbers $r_1, \cdots r_v$.

   Write down expressions for (a) "sum of squares for treatments (adjusted)" and the "sum of square for blocks (unadjusted)" [proof is not required]. Derive the expected values of both sums of squares. Looking at the expected values can you see any justification for the terms "adjusted" and "unadjusted" ?

   $[3 + 5 + 5 + 3) = 16]$

2. (a) When is a block design said to be connected ?

   Verify whether the following block design is connected.

   Block 1 : 1 1 2
   Block 2 : 3 3 4
   Block 3 : 2 5 4

   (b) When is a block design called "orthogonal" ?

   Show that if a connected block design is orthogonal then its incidence matrix ($N = ((n_{ij}))$) satisfies

   $$n_{ij} = r_i k_j / n.$$

   Is the design in Q 2(a) orthogonal ?

   Consider a connected design d with m factors. Let r^p denote the replication vector of the pth factor and N^{pq} denote the pth factor versus qth factor incidence matrix.

   When is the design d said to be orthogonal ? Obtain the condition for d to be orthogonal. Is a latin square design orthogonal ? justify.

   $[1 + 3 + (2 + 7 + 1) + (1 + 2 + 2) = 19]$

3. Construct a BIBD with the following parameters. [Either present all the blocks or describe the construction with proof].

   $v = 11, k = 5, \lambda = 2.$ [7]

4. Consider a row-column design with p rows, q columns, v treatments with replication numbers $r_1, \cdots r_v$.

   Write down the reduced normal equation for the treatment effects [proof is not required]. Under what condition this equation coincides with the reduced normal equation for the treatment effects for a block design ? Give an example with $p \neq v$. $[2 + 2 + 3 = 7]$

5. In a paper manufacturing factory, the percentage of hardwood concentration in raw pulp and the cooking time of pulp are being investigated for their effect on the strength of the paper.

   (a) Suppose two concentrations (C) and two cooking times (T) are used and Two observations are taken for each level combination. Explain what is meant by (i)the main effects of C and (ii) the interaction effect CT and how one can estimate them.

   (b) Construct a 3^4 experiment in blocks of size 9, so that the interactions ACD^2 and A^2BC are confounded with blocks. Find the other factorial effects which are confounded. See whether the block effect is orthogonal to the interaction CD.

   $[(3 + 4) + (5 + 2 + 3) = 17]$